THERMAL CONVECTION IN A VERTICAL ANNULAR TUBE

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It has been found that, for constant temperature difference between the halves of the annulus and circular motion of the liquid, $\overline{\mathrm{Nu}} = 0.125\ \mathrm{Ra}$. This equation is confirmed by experiment for Ra < 16000.

A closed tube in the form of an annulus is an essential part of many heat-exchangers, heat transfer resulting from free convection of a liquid contained in the tube. Study of the laws of convective heat transfer in such tubes for various boundary conditions is of considerable interest.

Let us examine a vertical annular tube (Fig. 1), filled with an incompressible viscous liquid, the liquid in the left half being maintained at the constant temperature $t_0 + t/2$, and that in the right half at $t_0 - t/2$. Since the density of the liquid in the right half will be greater than in the left, clockwise motion of the liquid is to be expected. If $R \gg r_0$, the velocity v of equilibrium circular motion of the liquid depends only on the distance r from the tube axis*. Then, in two symmetrically located volumes of liquid in the form of cylinders of radius r and length $Rd\theta$, the moment of the gravity forces is $\pi\rho g\beta t R^2 r^2 \sin\theta d\theta$, and on the whole annulus of diameter 2R and cross-sectional radius r the moment acting is $2\pi\rho g\beta t R^2 r^2$. This moment is balanced by the moment of the viscous friction forces, equal to $4\pi^2 R^2 r\,\eta\frac{dv}{dr}$.

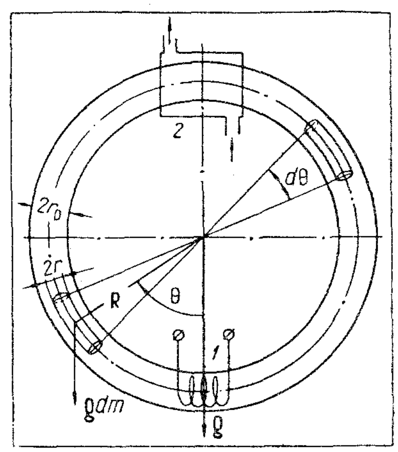

Fig. 1. Diagram of model: 1—heating coil; 2—cooler.

This condition gives $dv = -\frac{\rho g\beta t}{2\pi\eta}\,rdr$. Integrating this equation, we obtain the velocity profile

$$v = \frac{\rho g\beta t}{4\pi\eta}\,(r_0^2 - r^2). \tag{1}$$

For the mass and heat flow carried by convection from the left half of the annulus to the right, we have respectively

$$m = \frac{\rho^2 g\beta t}{8\eta}\,r_0^4;\quad q = cmt. \tag{2}$$

Since the liquid temperatures in the halves of the annulus are constant, the dimensionless heat flow over the cross section of the annulus has the significance of integral Nu. Taking as the unit of thermal flux $\lambda t r_0$, we obtain from (2) the following equation:

$$\overline{\mathrm{Nu}} = 0.125\,\mathrm{Ra}. \tag{3}$$

Relations (1)—(3) in our treatment are valid correct to a numerical factor for closed tubes of different shapes. Note by way of comparison that in [1] the natural circulation in a closed tube containing water, with a heat source underneath and convective heat transfer at its outer boundary, has been treated analytically. The dependence of the parameters was not established, but one would expect that it would be more complex than equation (3).

Tests were carried out on two models, consisting of glass or celluloid tubes bent into an annulus and filled with freshly boiled distilled water or benzine B = 70. Short nichrome electrical heater coils and junctions to copper lead-in wires were located in the liquid (Fig. 1). The supply to the heaters was stabilized. A cooler, fed from a flow ultrathermostat, was located on the annulus diametrically opposite the heater. In the celluloid model (R = 54.2 mm, r_0 = = 2.47 mm, a = 0.83 mm) copper-constantan thermocouple junctions of wire diameter 0.19 mm were soldered to 12 brass bands 2.5 mm wide and 0.2 mm thick, glued to the outer surface of the tube at various distances along its length. In the glass model (R = 144 mm, r_0 = 5.45 mm, a = 1.15 mm) the thermocouples were attached as in [2]. The thermocouple network comprised 24 stages, equally spaced along the tube, each containing 4 thermocouples. The thermo-

*We neglect change in velocity due to thermal expansion (or compression) in passing from one half of the annulus to the other.

couple readings were corrected for temperature drop in the model walls; thermal conductivity values for glass and celluloid were taken from tables. The convective heat loss of the model to the air was also taken into account. To reduce heat loss, the mean annulus temperature was generally arranged to be close to that of the air. Visualization of the liquid motion was accomplished by means of aluminum powder. In all tests the model annulus was set up in a vertical plane; measurements were carried out under steady conditions, obtained in 1–3 hr.

Qualitative visual and photographic examination showed that with heating strictly from below, two forms of liquid motion can be realized, either separately or together: a circular one whose direction is apparently determined by chance, and a cellular motion, in which heated liquid flows away from the heater, generally in currents of unequal area in the left and right sides of the annulus, and converging from both sides on the cooler, whence descending currents of cold liquid set out. The dividing plane of ascending and descending currents does not stabilize near the horizontal diameter of the annulus. If the model is rotated through 5–10° under low heating power, circulating motion is set up in the annulus, with a velocity profile resembling the Poiseuille case. For large heating power, currents at an angle to the tube axis occur here and there. Evidently, in this case superimposed on the circulating liquid motion we get a cellular motion, though on a smaller scale than in the case of heating from below. For angles of inclination of about 45°, the liquid is motionless in parts of the annulus situated below the heater and above the cooler; cellular motion is observed in the remainder of the tube.

In the case of cellular motion there are substantial differences in thermocouple readings within one zone (transverse temperature differences occur), and there are very complex, though reproducible distributions of zonal mean temperature along the annulus. For circulating liquid motion in the glass model at heater powers up to 10 w, no appreciable transverse temperature differences were observed; apart from small sections of the tube near the heater and cooler, temperatures in the left and right halves were practically constant, as the table shows. For greater heater power the scatter of thermocouple readings increases, but with circulating liquid motion it remains small in comparison with the differences between the mean temperatures in the two halves. Thus the isothermal premise adopted in the theoretical treatment of the half annulus arises naturally for the case of circulating liquid motion because of the concentrated heating and cooling at diametrically opposite points on the tube.

TABLE

Typical temperature distribution along annulus for circulating liquid motion (glass model and water, heater power 1.97 w, zones with coordinates θ = = 172.5 and 187.5° located inside the cooler)

Left half		Right half	
θ	τ, °C	θ	τ, °C
7.5	15.6	187.5	11.7
22.5	15.4	202.5	14.0
37.5	15.3	217.5	14.3
52.5	15.5	232.5	14.2
67.5	15.6	247.5	14.2
82.5	15.6	262.5	14.2
97.5	15.6	277.5	14.2
112.5	15.5	292.5	14.2
127.5	15.6	307.5	14.2
142.5	15.6	322.5	14.2
157.5	15.5	337.5	14.2
172.5	11.5	352.5	14.2

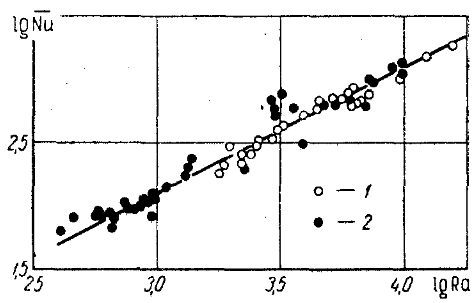

Fig. 2. Dependence of Nu on Ra: 1–glass model, tests with water; 2–celluloid model, tests with water and benzine.

Figure 2 gives the generalized experimental results on a logarithmic scale in the Ra number range from 350 to 16,000, for circulating liquid motion and model inclination up to 10°. The experimental points lie close to the theoretical line. Therefore, convective heat transfer in a vertical annular tube is described by equation (3) within the range of Ra numbers examined, for the case of circulating liquid motion.

For cellular liquid motion the temperatures of the halves are not constant, and the Nu number is a function of the radius R of the annulus and increases with Ra considerably more slowly than formula (3) indicates, the Ra number being calculated from the difference between the mean temperatures of the halves.

NOTATION

R and r_0–radii of annulus; a–model wall thickness; ρ, c, β, λ, $\varkappa$, η, ν–respectively, density, specific heat, coefficients of thermal expansion, thermal conductivity, diffusivity, and dynamic and kinematic viscosity at temperature t_0; $\mathrm{Ra} = \frac{g\beta}{\nu\varkappa} t r_0^3$–Rayleigh number; $\overline{\mathrm{Nu}} = \frac{q}{\lambda t r_0}$–integral value of Nusselt number; t–temperature differences of halves of model.

REFERENCES

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2. G. A. Ostroumov, Free Convection in Cases of Internal Flow [in Russian], GITTL, 1952.

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